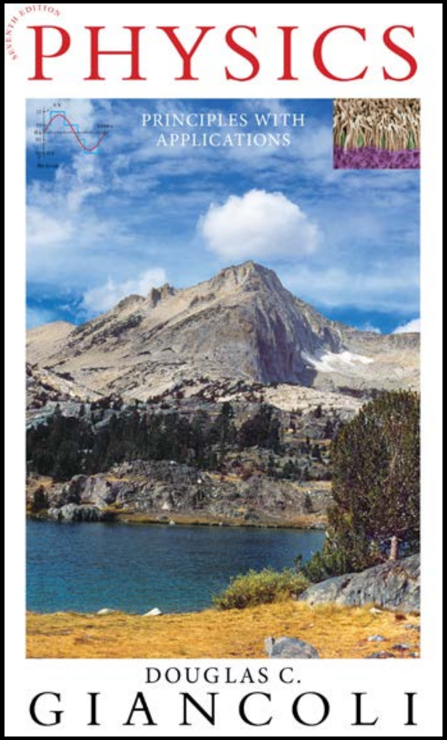

Lecture PowerPoints

Chapter 8

Physics: Principles with Applications, 7th edition

Giancoli

Chapter 8
Rotational Motion

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- Constant Angular Acceleration
- Rolling Motion (Without Slipping)
- Torque
- Rotational Dynamics; Torque and Rotational Inertia
- Solving Problems in Rotational Dynamics

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- Rotational Kinetic Energy
- Angular Momentum and Its Conservation
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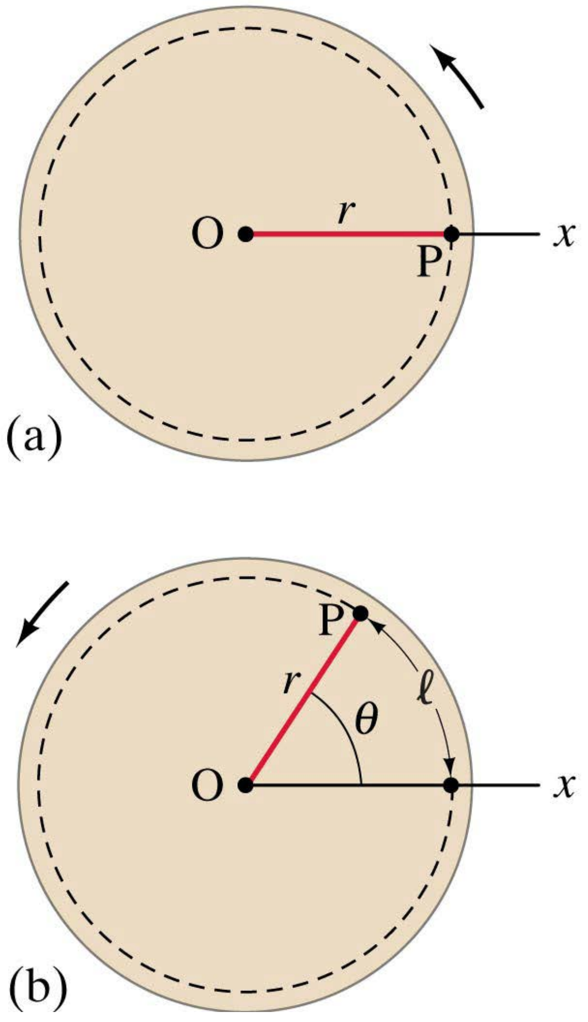
8-1 Angular Quantities

In purely rotational motion, all points on the object move in circles around the axis of rotation ("*O*"). The radius of the circle is *r*. All points on a straight line drawn through the axis move through the same angle in the same time. The angle θ in radians is defined:

$$\theta = \frac{\ell}{r}, \qquad \text{(8-1a)}$$

where *l* is the arc length.



8-1 Angular Quantities

Angular displacement:

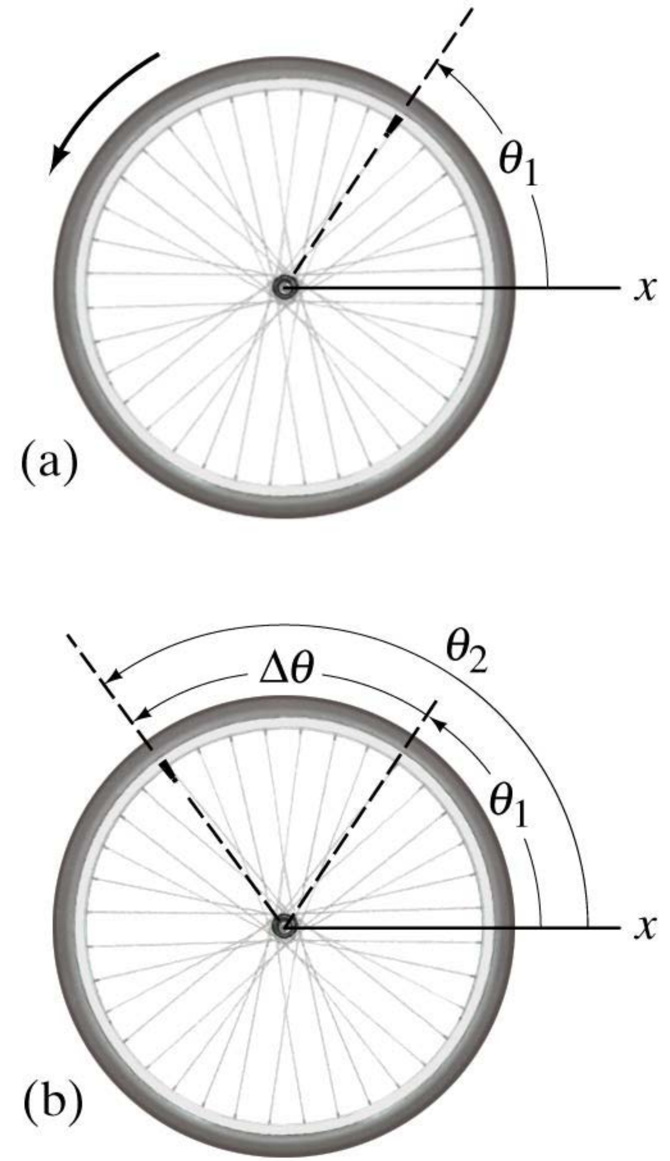
$$\Delta\theta = \theta_2 - \theta_1$$

The average angular velocity is defined as the total angular displacement divided by time:

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}, \qquad (8\text{-}2a)$$

The instantaneous angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}. \qquad (8\text{-}2b)$$



8-1 Angular Quantities

The angular acceleration is the rate at which the angular velocity changes with time:

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta\omega}{\Delta t} \qquad (8\text{-}3a)$$

The instantaneous acceleration:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}. \qquad (8\text{-}3b)$$

8-1 Angular Quantities

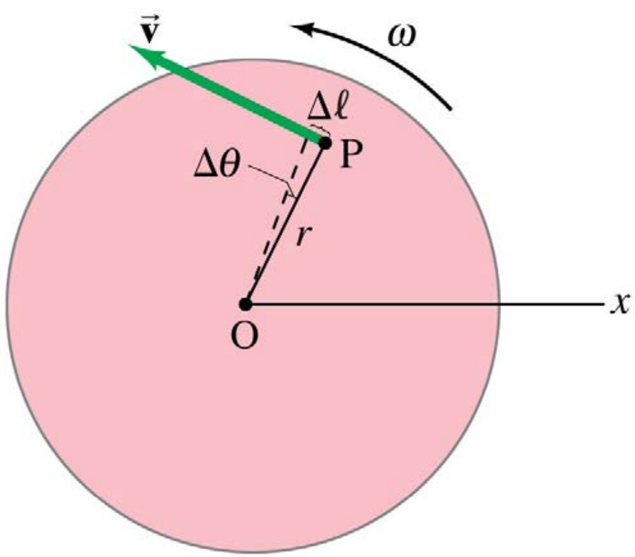
Every point on a rotating body has an angular velocity ω and a linear velocity v.

They are related:

$$v = \frac{\Delta \ell}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

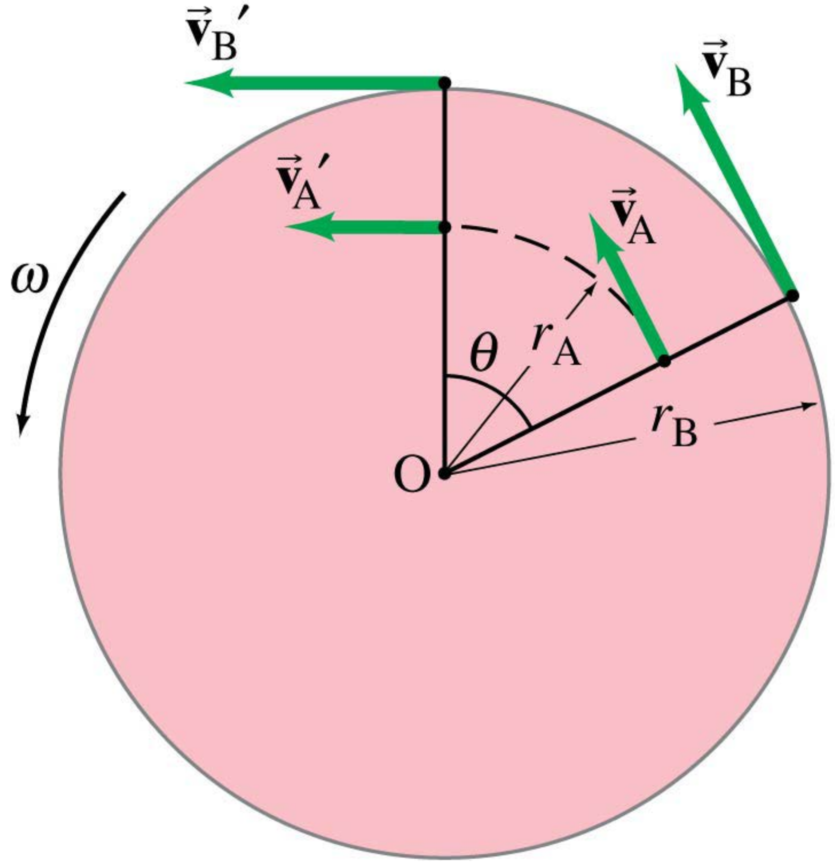
or (since $\Delta\theta/\Delta t = \omega$)

$$v = r\omega. \qquad (8\text{-}4)$$



8-1 Angular Quantities

Therefore, objects farther from the axis of rotation will move faster.



8-1 Angular Quantities

If the angular velocity of a rotating object changes, it has a tangential acceleration:

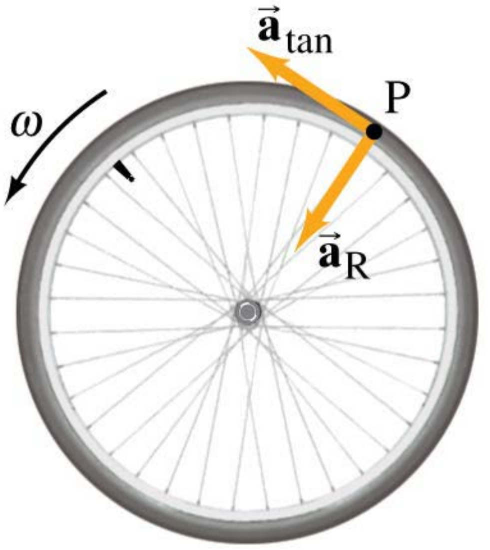
$$a_{\tan} = \frac{\Delta v}{\Delta t} = r\frac{\Delta \omega}{\Delta t}$$

or (using Eq. 8–3)

$$a_{\tan} = r\alpha. \qquad (8\text{-}5)$$

Even if the angular velocity is constant, each point on the object has a centripetal acceleration:

$$a_{\mathrm{R}} = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r. \qquad (8\text{-}6)$$



8-1 Angular Quantities

Here is the correspondence between linear and rotational quantities:

TABLE 8–1 Linear and Rotational Quantities

Linear	Type	Rotational	Relation‡
x	displacement	θ	$x = r\theta$
v	velocity	ω	$v = r\omega$
a_{tan}	acceleration	α	$a_{\text{tan}} = r\alpha$

‡ You must use radians.

8-1 Angular Quantities

The frequency is the number of complete revolutions per second:

$$f = \omega/2\pi$$

Frequencies are measured in hertz.

$$1 \text{ Hz} = 1 \text{ s}^{-1}$$

The period is the time one revolution takes:

$$T = \frac{1}{f}. \qquad (8\text{-}8)$$

8-2 Constant Angular Acceleration

The equations of motion for constant angular acceleration are the same as those for linear motion, with the substitution of the angular quantities for the linear ones.

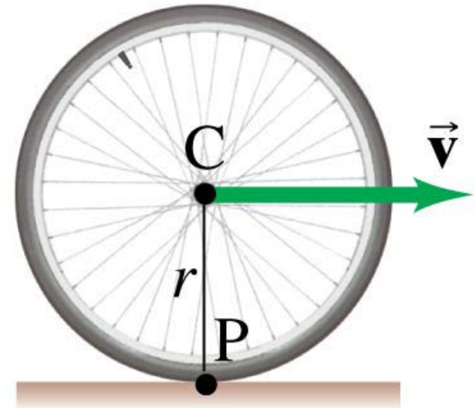
Angular	Linear		
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	[constant α, a]	**(8–9a)**
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$	[constant α, a]	**(8–9b)**
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$	[constant α, a]	**(8–9c)**
$\overline{\omega} = \dfrac{\omega + \omega_0}{2}$	$\overline{v} = \dfrac{v + v_0}{2}$	[constant α, a]	**(8–9d)**

8-3 Rolling Motion (Without Slipping)

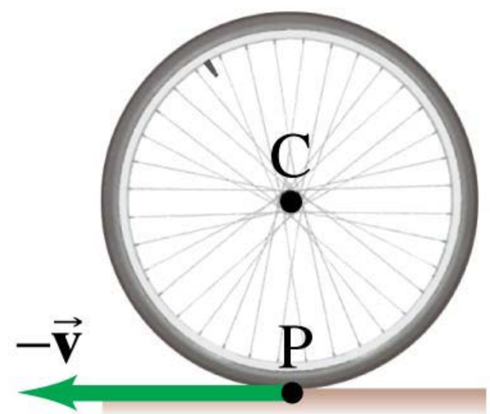
In (a), a wheel is rolling without slipping. The point P, touching the ground, is instantaneously at rest, and the center moves with velocity v.

In (b) the same wheel is seen from a reference frame where C is at rest. Now point P is moving with velocity –v.

Relationship between linear and angular speeds: $v = r\omega$



(a)

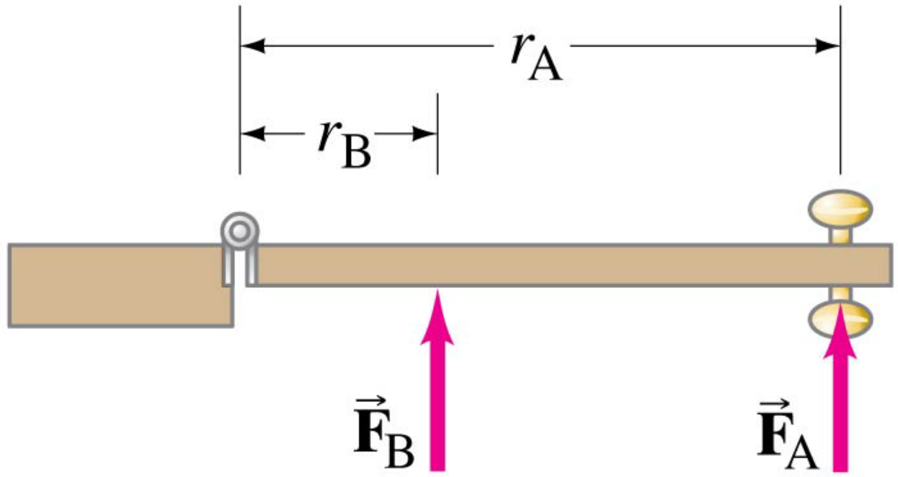


(b)

8-4 Torque

To make an object start rotating, a force is needed; the position and direction of the force matter as well.

The perpendicular distance from the axis of rotation to the line along which the force acts is called the lever arm.



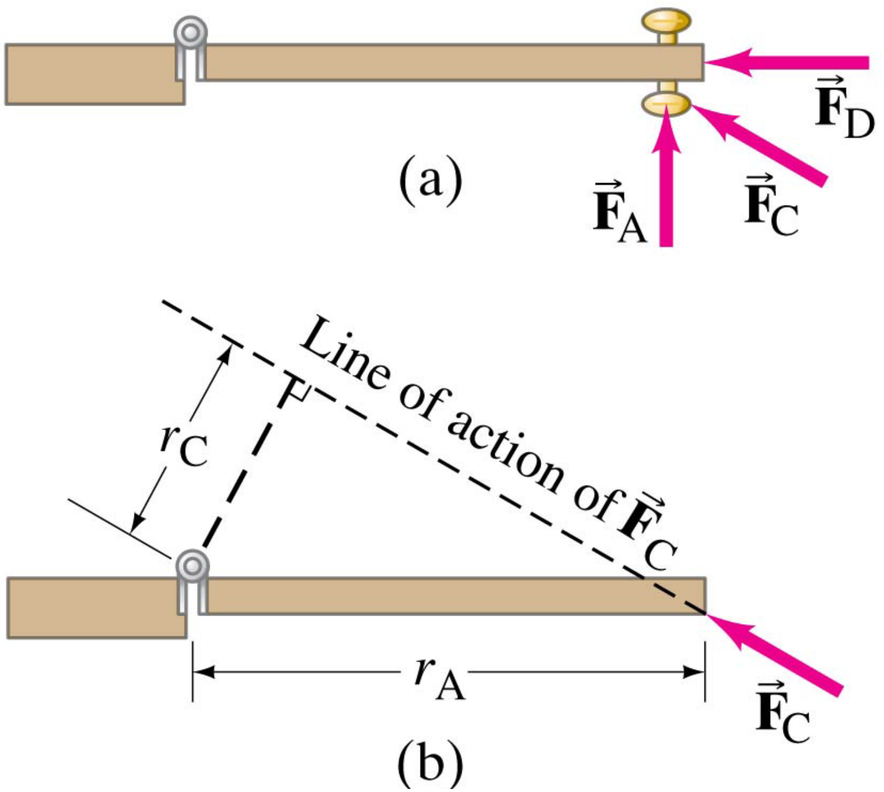
8-4 Torque

A longer lever arm is very helpful in rotating objects.

(a) (b)

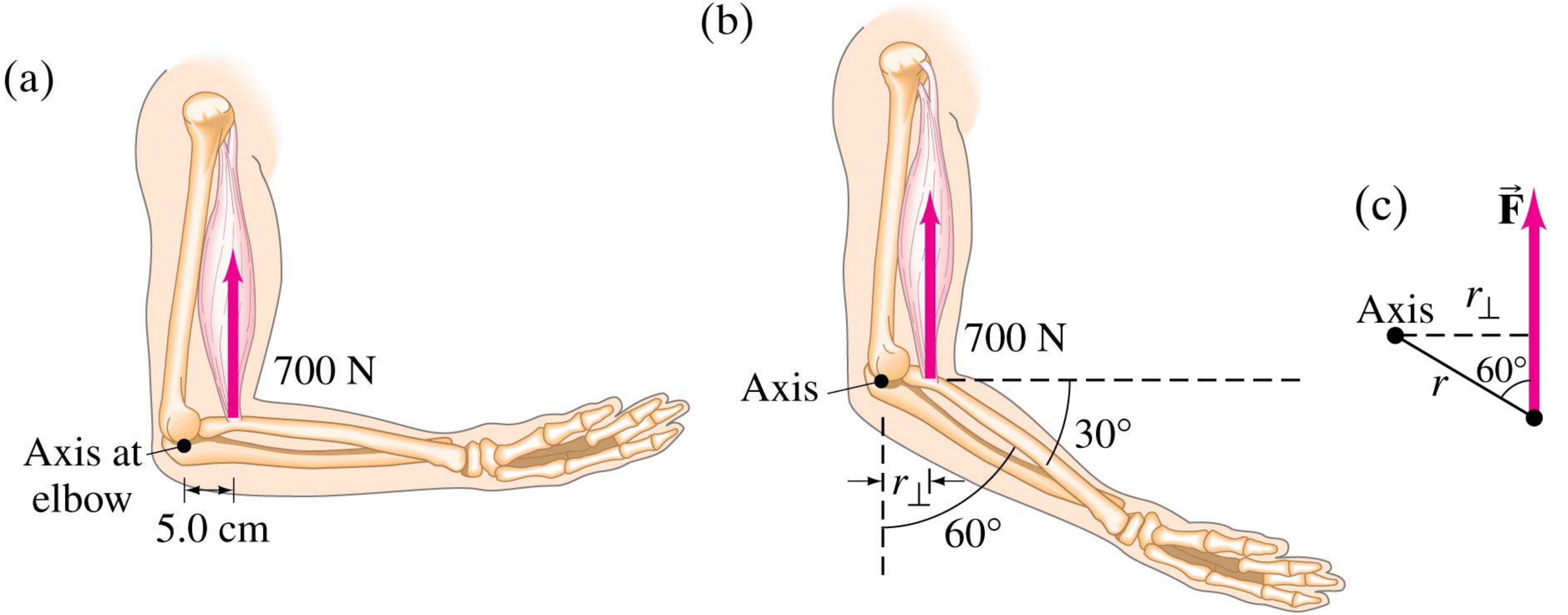
8-4 Torque

Here, the lever arm for F_A is the distance from the knob to the hinge; the lever arm for F_D is zero; and the lever arm for F_C is as shown.



8-4 Torque

The torque is defined as: $\tau = r_{\perp} F$ (8-10a)



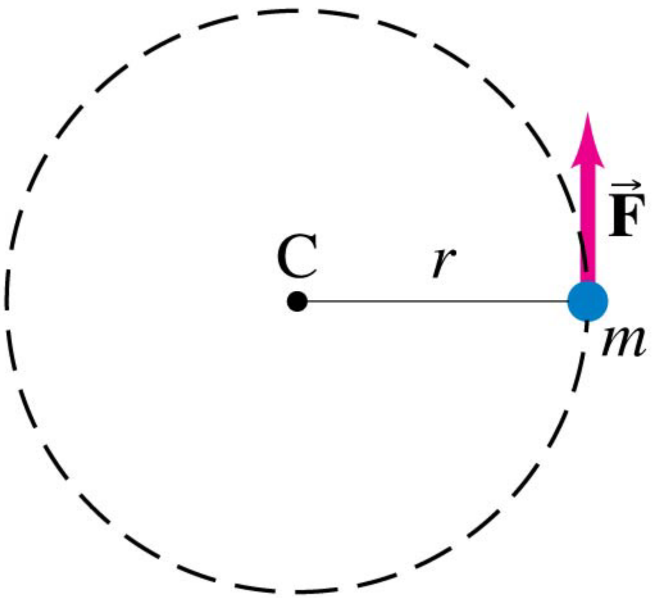
8-5 Rotational Dynamics; Torque and Rotational Inertia

Knowing that $F = ma$, we see that $\tau = mr^2\alpha$

This is for a single point mass; what about an extended object?

As the angular acceleration is the same for the whole object, we can write:

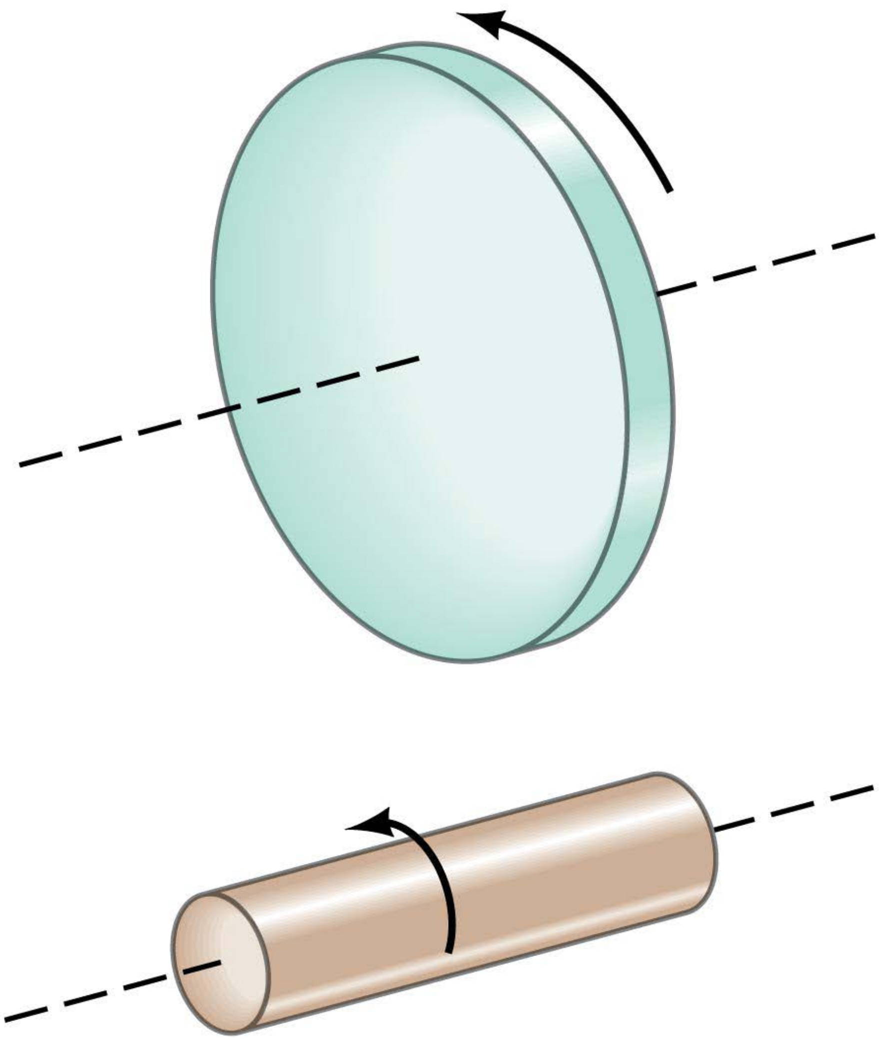
$$\Sigma\tau = \left(\Sigma mr^2\right)\alpha \qquad (8\text{-}12)$$



8-5 Rotational Dynamics; Torque and Rotational Inertia

The quantity $I = \Sigma mr^2$ is called the rotational inertia of an object.

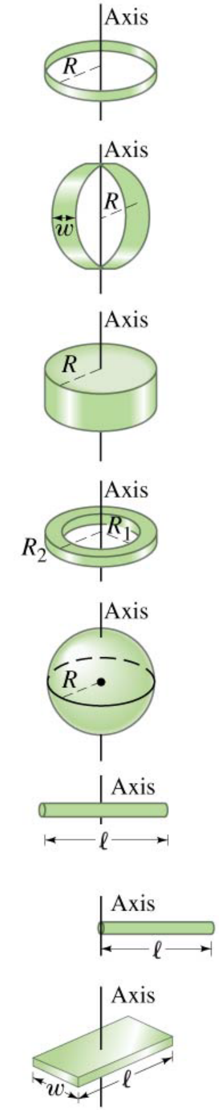
The distribution of mass matters here—these two objects have the same mass, but the one on the left has a greater rotational inertia, as so much of its mass is far from the axis of rotation.



8-5 Rotational Dynamics; Torque and Rotational Inertia

The rotational inertia of an object depends not only on its mass distribution but also the location of the axis of rotation—compare (f) and (g), for example.

	Object	Location of axis	Moment of inertia
(a)	**Thin hoop,** radius R	Through center	MR^2
(b)	**Thin hoop,** radius R width w	Through central diameter	$\frac{1}{2}MR^2 + \frac{1}{12}Mw^2$
(c)	**Solid cylinder,** radius R	Through center	$\frac{1}{2}MR^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius R	Through center	$\frac{2}{5}MR^2$
(f)	**Long uniform rod,** length ℓ	Through center	$\frac{1}{12}M\ell^2$
(g)	**Long uniform rod,** length ℓ	Through end	$\frac{1}{3}M\ell^2$
(h)	**Rectangular thin plate,** length ℓ, width w	Through center	$\frac{1}{12}M(\ell^2 + w^2)$



8-6 Solving Problems in Rotational Dynamics

1. Draw a diagram.
2. Decide what the system comprises.
3. Draw a free-body diagram for each object under consideration, including all the forces acting on it and where they act.
4. Find the axis of rotation; calculate the torques around it.

8-6 Solving Problems in Rotational Dynamics

5. Apply Newton's second law for rotation. If the rotational inertia is not provided, you need to find it before proceeding with this step.

6. Apply Newton's second law for translation and other laws and principles as needed.

7. Solve.

8. Check your answer for units and correct order of magnitude.

8-7 Rotational Kinetic Energy

The kinetic energy of a rotating object is given by

$$\text{KE} = \Sigma(\tfrac{1}{2} mv^2)$$

By substituting the rotational quantities, we find that the rotational kinetic energy can be written:

$$\text{rotational KE} = \tfrac{1}{2} I\omega^2. \qquad (8\text{-}15)$$

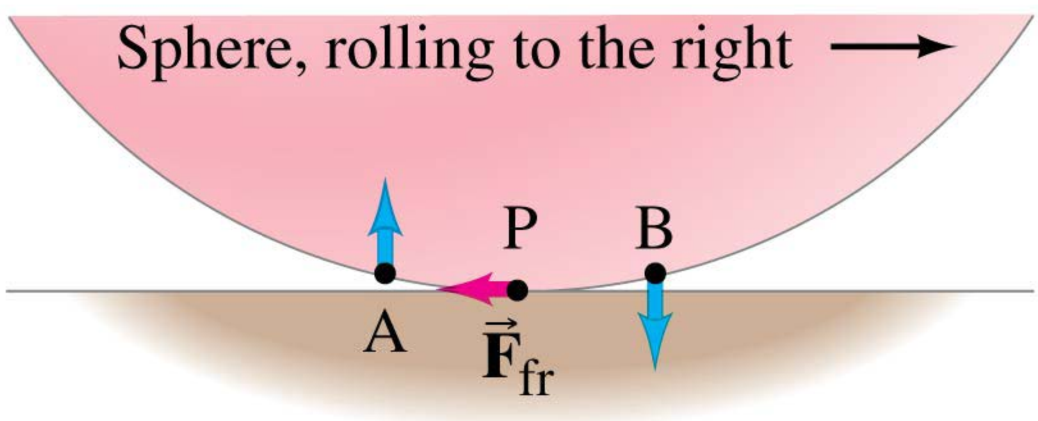
A object that has both translational and rotational motion also has both translational and rotational kinetic energy:

$$\text{KE} = \tfrac{1}{2} Mv_{\text{CM}}^2 + \tfrac{1}{2} I_{\text{CM}}\,\omega^2 \qquad (8\text{-}16)$$

8-7 Rotational Kinetic Energy

When using conservation of energy, both rotational and translational kinetic energy must be taken into account.

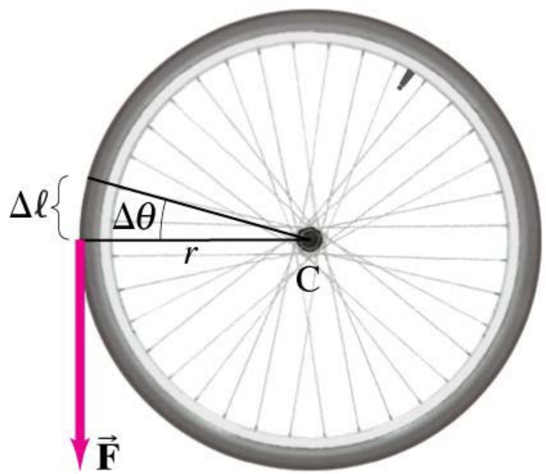
All these objects have the same potential energy at the top, but the time it takes them to get down the incline depends on how much rotational inertia they have.



8-7 Rotational Kinetic Energy

The torque does work as it moves the wheel through an angle θ:

$$W = \tau \, \Delta\theta \qquad (8\text{-}17)$$



8-8 Angular Momentum and Its Conservation

In analogy with linear momentum, we can define angular momentum *L*:

$$L = I\omega \qquad (8\text{-}18)$$

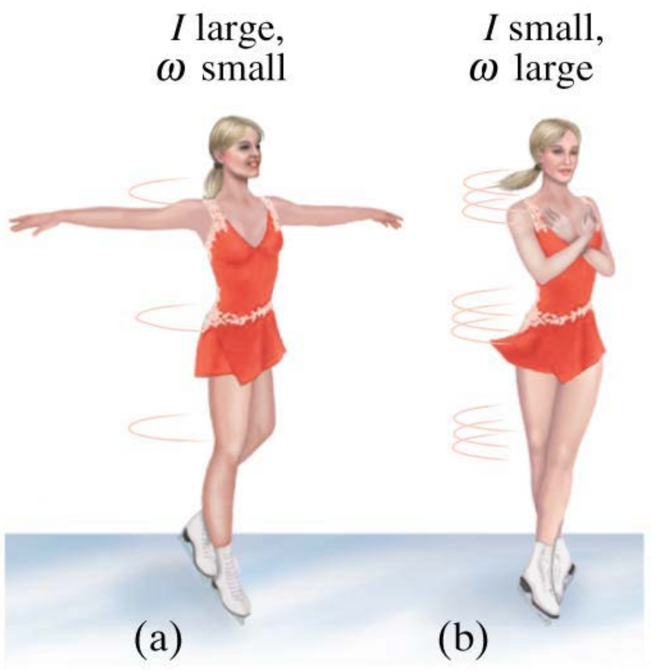
We can then write the total torque as being the rate of change of angular momentum.

If the net torque on an object is zero, the total angular momentum is constant.

$$I\omega = I_0\omega_0 = \text{constant}$$

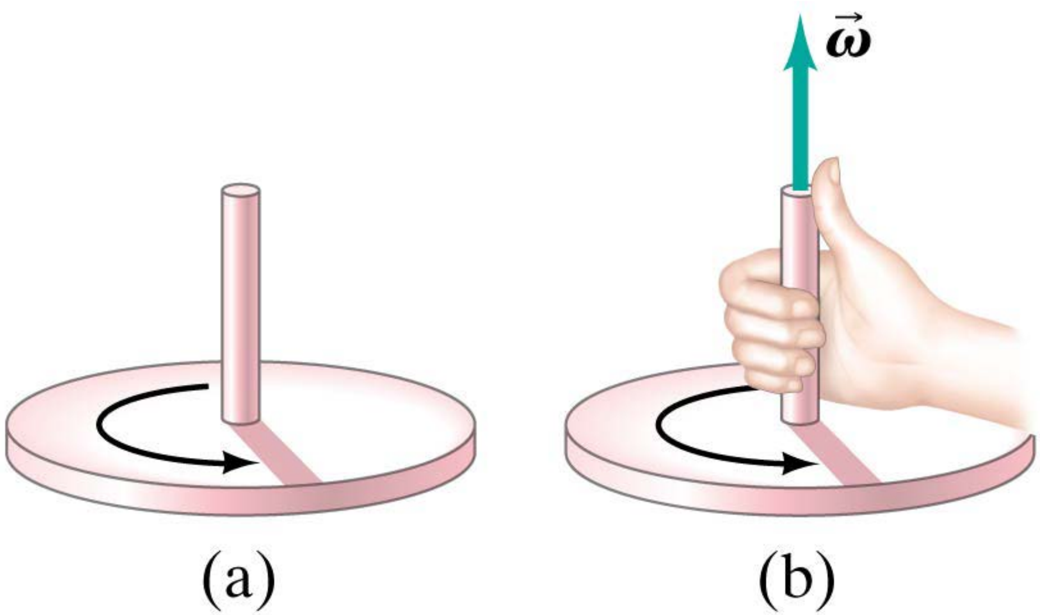
8-8 Angular Momentum and Its Conservation

Therefore, systems that can change their rotational inertia through internal forces will also change their rate of rotation:



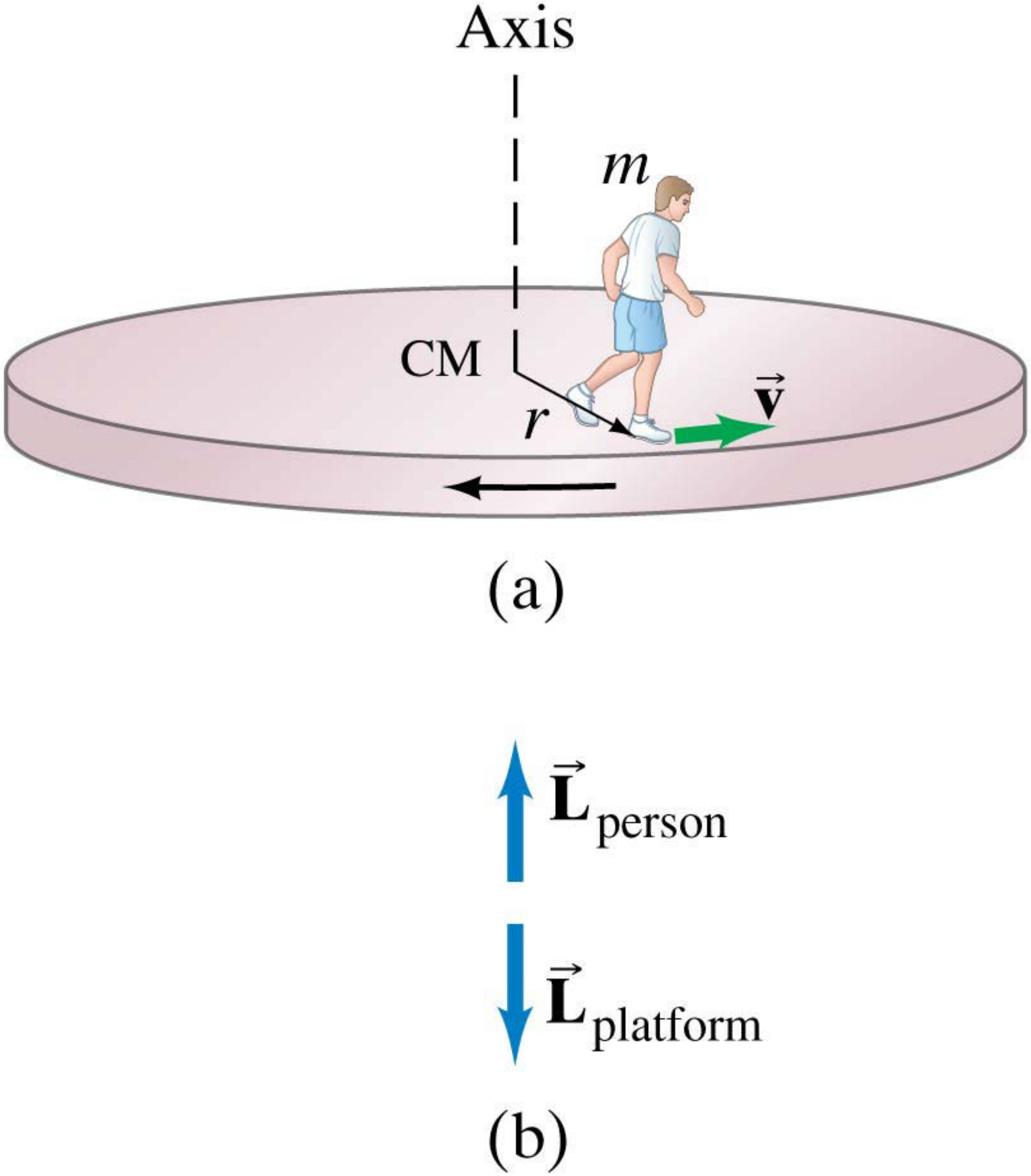
8-9 Vector Nature of Angular Quantities

The angular velocity vector points along the axis of rotation; its direction is found using a right hand rule:



8-9 Vector Nature of Angular Quantities

Angular acceleration and angular momentum vectors also point along the axis of rotation.



Summary of Chapter 8

- Angles are measured in radians; a whole circle is 2π radians.
- Angular velocity is the rate of change of angular position.
- Angular acceleration is the rate of change of angular velocity.
- The angular velocity and acceleration can be related to the linear velocity and acceleration.

Summary of Chapter 8

- The frequency is the number of full revolutions per second; the period is the inverse of the frequency.
- The equations for rotational motion with constant angular acceleration have the same form as those for linear motion with constant acceleration.
- Torque is the product of force and lever arm.
- The rotational inertia depends not only on the mass of an object but also on the way its mass is distributed around the axis of rotation.

Summary of Chapter 8

- The angular acceleration is proportional to the torque and inversely proportional to the rotational inertia.
- An object that is rotating has rotational kinetic energy. If it is translating as well, the translational kinetic energy must be added to the rotational to find the total kinetic energy.
- Angular momentum is $L = I\omega$
- If the net torque on an object is zero, its angular momentum does not change.